Combine figures 3 and 4

~~thank James Binney~~

first submit to astoph and then monthly notices

then send copy to scott saying its been sumbitted (improved the treatment) CC ostriker and philip

~~I also have one question. When you make your plots of the heating, you integrated over time. What is the starting time in this integral? Is it the age of the universe? I imagine that probably the answer is not very sensitive to the time interval / start time you use, but just mention which equations you integrate over what time interval in the paper to create the figures.  I am just asking because the FDM fluctuations show up at around the collapse-time / free-fall time of the halo, i.e. when you have multiple shell crossing, before that the DM distribution is smooth.~~

Section 4.3

~~\* If heating is dominant in driving the evolution of σD then, since they predict the heating rate depends ...    what are you referring to by "they" over here?~~

~~Latex / typesetting:~~

~~\* fix m\_{acc} to m\_{\rm acc} in some places because I see both styles in the paper~~

~~\* Equation 30. lhs is missing ")"~~

~~\* in the text, be consistent with "figure N" vs "Figure N"~~

Although highly successful on cosmological scales, Cold Dark Matter (CDM) models predict unobserved over-dense <i>cusps</i> in dwarf galaxies and overestimate their formation rate. We consider an ultra-light axion-like scalar boson which promises to reduce these observational discrepancies at galactic scales. The model, known as Fuzzy Dark Matter (FDM), avoids cusps, suppresses small-scale power, and delays galaxy formation via macroscopic quantum pressure. We compare the substructure of galactic dark matter halos comprised of ultra-light axions to conventional CDM results. Besides self-gravitating subhalos, FDM includes additional substructure in the form of non-virialized over-dense wavelets formed by quantum interference patterns which provide a more efficient source of heating to galactic discs than do subhalos. We find that, in the solar neighborhood, wavelet heating is sufficient to give the oldest disc stars a velocity dispersion in excess of 30 km s<sup>-1</sup> within a Hubble time if energy is not lost from the disc. Furthermore, we calculate the radius-dependent velocity dispersion and corresponding scale height caused by the heating of dynamical substructure in both CDM and FDM with the determination that these effects will produce a flaring that terminates the Milky Way disc at 15 - 20 kpc. Although the source of thickened discs is not known, the heating due to perturbations caused by dark substructure cannot exceed the total disc velocity dispersion. Therefore, this work provides a lower bound on the FDM particle mass of m<sub>a</sub> > 0.63 × 10<sup>-22</sup> eV. Furthermore, FDM wavelets with particle mass m<sub>a</sub> ∼ 0.7 × 10<sup>-22</sup> eV. should be considered a viable mechanism for producing the observed disc thickening with time.

Although highly successful on cosmological scales, Cold Dark Matter (CDM) models predict unobserved over-dense \squote{cusps} in dwarf galaxies and overestimate their formation rate. We consider an ultra-light axion-like scalar boson which promises to reduce these observational discrepancies at galactic scales. The model, known as Fuzzy Dark Matter (FDM), avoids cusps, suppresses small-scale power, and delays galaxy formation via macroscopic quantum pressure. We compare the substructure of galactic dark matter halos comprised of ultra-light axions with masses $\poweV{-24} \leq m\_a \leq \poweV{-18}$ to conventional CDM results. Besides self-gravitating subhalos, FDM includes additional substructure in the form of non-virialized over-dense wavelets formed by quantum interference patterns which provide a more efficient source of heating to galactic discs than do subhalos. We find that, in the solar neighborhood, wavelet heating is sufficient to give the oldest disc stars a velocity dispersion in excess of \SI{30}{\kilo\meter\per\second} within a Hubble time if energy is not lost from the disc.

Furthermore, we calculate the radius-dependent velocity dispersion and corresponding scale height caused by the heating of dynamical substructure in both CDM and FDM with the determination that these effects will produce a flaring that terminates the Milky Way disc at \SIrange{15}{20}{\kilo \parsec}. Although the source of thickened discs is not known, the heating due to perturbations caused by dark substructure cannot exceed the total disc velocity dispersion. Therefore, this work provides a lower bound on the FDM particle mass of $m\_a > \SI{0.63 e-22}{\electronvolt}$. Furthermore, FDM wavelets with particle mass $m\_a \sim \SI{0.7e-22}{\electronvolt}$ should be considered a viable mechanism for producing the observed disc thickening with time.

Although highly successful on cosmological scales, Cold Dark Matter (CDM) models predict unobserved over-dense *cusps* in dwarf galaxies and overestimate their formation rate. We consider an ultra-light axion-like scalar boson which promises to reduce these observational discrepancies at galactic scales. The model, known as Fuzzy Dark Matter (FDM), avoids cusps, suppresses small-scale power, and delays galaxy formation via macroscopic quantum pressure. We compare the substructure of galactic dark matter halos comprised of ultra-light axions to conventional CDM results. Besides self-gravitating subhalos, FDM includes additional substructure in the form of non-virialized over-dense wavelets formed by quantum interference patterns which provide a more efficient source of heating to galactic discs than do subhalos. We find that, in the solar neighborhood, wavelet heating is sufficient to give the oldest disc stars a velocity dispersion in excess of 30 km s-1 within a Hubble time if energy is not lost from the disc. Furthermore, we calculate the radius-dependent velocity dispersion and corresponding scale height caused by the heating of dynamical substructure in both CDM and FDM with the determination that these effects will produce a flaring that terminates the Milky Way disc at 15 - 20 kpc. Although the source of thickened discs is not known, the heating due to perturbations caused by dark substructure cannot exceed the total disc velocity dispersion. Therefore, this work provides a lower bound on the FDM particle mass of ma > 0.63 × 10-22 eV. Furthermore, FDM wavelets with particle mass ma ∼ 0.7 × 10-22 eV. should be considered a viable mechanism for producing the observed disc thickening with time.

6 + 4 + 6 + 3 = 19

Scientific Editor: White, Simon  
  
  
**Assistant Editor's Comments:  
Please ensure that all textual labels in figures are at least as large as the caption text; any smaller and they become too difficult to read.**  
  
  
Reviewer's Comments:  
Reviewer: 1  
  
Comments to the Author  
This paper investigates the effects of Cold Dark Matter (CDM) and Fuzzy Dark Matter (FDM) fluctuations on the dynamical heating of the Milky Way (MW) disc stars, and the associated increase in the stellar velocity dispersion.  
Using these predictions jointly with the observed velocity dispersion of the stars from the MW thick disc,  
the authors provide a lower bound on the mass on the FDM particles that would be compatible with this mechanism.  
The paper is organised as follows:  
\* Sect. 1 is thorough and detailed introduction that presents the main differences between the CDM and FDM paradigms.  
\* Sect. 2 presents the author's model of the MW stellar discs, and summarises the different mechanisms that have been put forward in the literature to explain the thick disc.  
\* Sect. 3 reviews the mechanisms of heating of the stellar disc caused by DM fluctuations, in particular within the CDM and FDM frameworks.  
In that section, the authors also apply their derived expression to predict the heating of the stellar disc and the associated increase in the velocity dispersion.  
\* Sect. 4 is a discussion of their results, placing a constraint on the mass of the FDM and comparing with similar recent works.  
  
++++++++++  
  
The results put forward in that work are both interesting and timely.  
They are undoubtedly a needed first step towards placing more and more observational constraints on the FDM model,  
hoping that it will end up solving the various hurdles encountered by CDM on small scales (see Introduction).  
While I am convinced that these results deserve to be published, I feel that in its present form, the present paper suffers from being disorganised in some places, making the reading occasionally unnecessarily confusing.  
I listed below my main questions/remarks on the paper.  
I would like the authors to pay a particular attention to points 1/, 5/, and 14/, where I made suggestions to improve the readability and structure of the paper, hoping to make it easier to follow.  
  
  
1/ Pages 1 and 2:  
Following my suggestion of rearranging some of the sections, I believe that the Introduction would benefit from exhibiting more clearly its structure, e.g. by introducing subsections, or rearranging the order of some paragraphs

(ostriker is doing this)

~~2/ Page 3:  
"Therefore, these results are able to put a lower bound [...] producing thick discs."  
-->  
At this stage, the authors should clarify quantitatively why one should expect a lower bound on the mass of the FDM particles from the velocity dispersion constraint  
Indeed, it can sound counter-intuitive that heavier FDM particles are less efficient at heating the disc.~~

*~~(add this in words)~~*

*~~Lower mass particles → larger wavelength → larger perturbation → more relaxation for given density.~~*   
  
~~3/ Page 4:  
"The period of disc oscillations [...] perturbing interactions."  
-->  
Here, the authors could clarify the adiabatic argument, and in particular in which direction it goes.  
Namely, they should mention explicitly that perturbations that are slower than vertical oscillation period cannot affet the stellar vertical energy.~~

*~~(be very explicit reference appendix)~~*   
  
~~4/ Page 4:  
"Under the assumption ... flaring of actual discs."  
-->  
The calculations of the increase of \sigma\_D are not yet presented at this stage.  
The authors should refer to the correct section where it is detailed.~~  
  
~~5/ Page 4, Section 3:  
This section is the most difficult section to follow, in particular because it can feel a bit disorganised, e.g. jumping from CDM to FDM, from theory to applications, etc.  
I would suggest the authors to re-order/re-organise these sections to improve the flow of the reading.  
Here are some suggestions [titles should be improved]:  
This entire section could be split in two sections:  
3: Theory of heating  
4: Application to the MW  
with the following subsections  
3. Theory of dynamical heating  
3.1. Heating due to transits  
3.2. Heating due to subhalos  
3.3. Heating due to wavelets  
3.2. Modelling the adiabatic changes in the disc  
4. Application  
4.1. Heating in CDM  
4.1.1. Assumptions: (Mass Function + Tidal disruption)  
4.1.2. Results: (Disc heating + plots)  
4.2. Heating in FDM  
4.2.1. Assumptions: (Mass Function + Tidal disruption)  
4.2.2. Results: (Plots)  
4.3. Heating of the disc (Plots)  
  
All of these are suggestions, but I feel that the paper would benefit from picking a better order to present the ideas/results.~~  
  
~~6/ Page 5:  
"The initial shape of the subhalo mass function is assumed to be spatially invariant, [...] decoupled"  
-->  
Are there any justifications/references for this assumption?  
Indeed, one could be could concerned that this separability assumption might be play an important role as one considers the effect of the dynamical heating as a function of the radius within the disc.~~

*~~(given the self-similar nature of the CDM power spectrum smaller halos are incorporated in larger halos ) (look at NFW)~~*  
~~7/ Page 6:  
"We note that Eq. (24) is consistent ..."  
-->  
Under its present form, this paragraph is quite obscure.  
Moreover, why would one expect the properties of the heating mechanism to satisfy any scaling properties?~~~~8/ Page 10:  
Rather than giving the tables 1 and 2, it could be clearer and more readable to represent these results using a (single?) plot.~~

*~~(remove the plots)~~* ~~9/ Page 11, Figs. 5 and 6:  
If available, it would be very informative to add on Figs. 5 and 6, the observed scale height of the thick disc of the MW.  
Of course, as emphasised by the authors, this comparison has to be taken with a grain of salt, as DM is not the only mechanism expected to heat up the disc vertically, i.e. with their DM calculations, the authors only provide a upper bound on the total heating of the disc.~~

*~~(do this and expend the plot to 2 kpc )~~*

*~~(because of interactions with the bar, the inner parts of this plot will not be reliable)~~*

*~~(however the additional thickening interior to the solar radius is observed)~~*

**10/ Page 12:  
Equation below (40): m\_a > 0.63 x 10^(-22) eV  
Quoting a final result with two figures after the decimal sounds over-optimistic given all the approximations involved in the derivation of that constraint?  
At this stage, the authors could also briefly summarise the main source of uncertainties coming into their confidence levels.**

**JUST LIST THE UNCERTAINTIES**   
  
~~11/ Page 12, Fig. 7  
"under the assumption that FDM substructure ..."  
-->  
The phrasing can be a bit misleading, as "FDM substructure" could be confused with only accounting for the "FDM subhalos" that are inefficient for such small r (see Fig. 2).  
Here, I believe that the authors have accounted for both sources of FDM heating (i.e. subhalos+wavelets).~~

~~(replace with “density fluctuations”)~~   
  
~~12/ Page 12, Fig. 7  
If available, it would be very informative to add on Fig. 7, the observed velocity dispersion of the stars in the thick disc.  
Of course, as emphasised by the authors, this comparison has to be taken with a grain of salt, as DM is not the only mechanism expected to heat up the disc vertically, i.e. with their DM calculations, the authors only provide a upper bound on the total heating of the disc.  
[In that caption, the authors should also repeat what the gray lines represent.]~~

*~~I though I did do this (ah the gray lines).~~*  
  
~~13/ Page 12:  
"Multiple sources of heating may combine to produce a lower effective bound."  
-->  
I am not sure that I understand this statement.  
Naively, one would expect that the more mechanisms one accounts, the faster the disc is going to heat.  
In order to explain the difference in the time scaling of the heating, what one needs is a mechanism that has a different time scaling than the FDM one?~~

*~~If there are other mechanisms than in order to overheat, this meachanism must be less effective thus giving a stricter bound in order to not overheat.~~*   
  
~~14/ Page 13:  
"4.4 Conclusion and Comparisons with other current work..."  
-->  
In order to make the paper more readable, I believe that this section should be split up in two parts:  
\* 4.4: Comparison with other current work:  
--> In that subsection, the authors would present their current discussion of two related papers: Amorisco&Loeb(2018) and Bar-Or et al. (2018)  
\* 5.: Conclusion  
In that section, the authors would summarise the main results of their investigation, and present possible future works.~~  
 **15/ Page 14:  
One of the limitation of the calculation presented in this Appendix, is the assumption that stars are on circular orbit (i.e. orbits with J\_r=0).  
What is the impact of stars being on eccentric orbits, i.e of stars being able to visit different r?  
Can this easily be accounted for?  
Even if not accounted for, I believe that this limitation should be mentioned/discussed, and ideally its impact roughly estimated?**

**NO CHANGE TO PAPER JUST DRAFT RESPONSE**

***(we take the conventional assumptions of treating disc stars as having perturbed circular orbits with small epicycle motions and vertical motions compared to the radius of their orbits. Cite Binney and Tremaine. Ben-Or and Tremaine take random orbits (cite) ) Look at the GMCs how they deal with comoving coordinates. Note that this is the standard approach. (Will be second order in H/r ~ 0.1 so effect is ~ 0.01 assuming that epicycles are on order of the scale height)***   
  
~~16/ Page 14, Eq. (49a):  
The definition of the vertical action seems a bit unsual, in particular, the absence of a 1/(2 \pi) prefactor (to compare with Eq. (3.192) of Binney&Tremaine 2008).  
Is there a misprint there?~~

*~~(look at the correct convention)~~*  
Minor remarks:  
  
~~\* Page 2:  
"a power law divergence in the number of small subhalos" --> REF?~~  
~~\* Page 4, Eq. (10)  
The authors could provide some seminal references on heating due to (linearised) impulsive encounters, this is no new calculation.~~  
  
~~\* Page 5:  
"the same form as the free halo mass [...] Press-Schechter formalism" --> REF?~~  
  
~~\* Page 6:  
"the rate of heating given by eqution (17) depends..." --> [I believe that you rather want to refer to Eq. (24).]~~  
  
~~\* Page 11, Figure 5:  
"heating due to FDM wavelets"  
--> To justify not accouting for FDM subhalos, the authors could refer explicitly to Fig. 2, where this mechanism is shown to be inefficient for r<15kpc.~~  
  
~~\* Page 12:  
"with age as \hat{J}\_{z} \propto ..."  
--> What is $\hat{J}\_z$ compared to J\_z?~~  
  
Unfortunately, there are also numerous misprints in the current version that sometimes made the reading a bit difficult.  
~~\* Page 2:  
then 1kpc --> than 1 kpc  
\* Page 2:  
milky-way-like --> Milky-Way-like  
\* Page 2:  
though the entire --> through the entire  
\* Page 3:  
r\_0~3.2 --> [Missing an unit]  
\* Page 3:  
the disc with pressure supporting --> [Something is missing in this sentence.]  
\* Page 4:  
where $P$ is the --> [There should be no alinea]~~  
~~\* Page 4:  
lateral oscillation --> vertical oscillations~~  
~~\* Page 4:  
adorementiond --> aforementioned~~  
~~\* Page 4:  
Eq. (12a): v\_p has not been defined~~  
~~\* Page 5:  
Eq. (14a): \sigma\_H has not been defined~~  
~~\* Page 5:  
in to two --> in two~~  
~~\* Page 6:  
that entire mass --> that the entire mass~~  
~~\* Page 6:  
interference patters --> interference patterns~~  
~~\* Page 6:  
de-Broglie --> de Broglie~~  
\* Page 8:  
~~Figure 2:  
1.75 \times 10^(-22) eV.  [Remove the final "."]~~  
~~\* Page 9:  
though to be small --> thought to be small~~  
~~\* Page 10:  
G. van Dokkum et al. (2013) --> [Missing parentheses around this reference]~~  
~~\* Page 12:  
a old thick disc --> an old thick disk~~  
~~\* Page 12:  
large relatively large --> relatively large~~  
~~\* Page 12:  
action of starts at --> action of stars~~  
~~\* Page 14:  
, gravitational acceleration is -->, the gravitational acceleration is~~  
~~\* Page 14:  
value Hamiltonian --> value of the Hamiltonian  
\* Page 15:~~  
~~then then --> then~~  
~~\* Page 15:  
before Eq. (69): where, --> where~~  
~~\* Page 15:  
some short of --> some sort of~~